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Theory of Travelling Salesman Problem

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***Abstract*—Some methods to solve the Traveling Salesman Problem (TSP) are examined in this paper. This paper shows some theories that are related in solving the TSP. Karl Menger was the first to study about the general form of the TSP. Then, Hassler, Whitney and Merill promote the issue. Schrijver explained that there is a definite portrayal about the connection amongst Menger and Whitney and the development of the TSP. The TSP is typical of an extensive class of "hard" improvement issues that have captivated mathematicians and PC researchers for a considerable length of time. Symmetric traveling salesman problem (sTSP), asymmetric traveling salesman problem (aTSP), and multi traveling salesman problem (mTSP) are the class of the TSP. The motivation behind this paper is to demonstrate the mathematical theory and show for the traveling salesman problem. Towards the end of the paper, the mathematical theory of traveling salesman problem will be studied and stated clearly. Some conclusions are presented also.**

Keywords: **Traveling Salesman Problem, theory, TSP**

I. INTRODUCTION

In the 18th century, the Traveling Salesman Problems (TSP) were studied by mathematician named Sir William Rowan Hamilton and by mathematician named Thomas Penyngton Kirkman [1]. TSP is defined as which, given N cities in the Euclidean plane, the issue is to discover a tour sequence with a minimum tour length beginning at some city and going by each city once and then coming back to the start point [2]. Since TSP lies in the class of NP-complete problem, one cannot locate the precise shortest path in a brief timeframe except for small-sized problems [3]. Traveling Salesman Problem is a standout amongst the most contemplated problems since it is easy to understand but that as it may, hard to tackle. Urban areas are represented by vertices and the separations between two places are represented by weighted edge in a complete weighted undirected chart G(V,E). TSP is to locate the minimized Hamilton cycle that begins from a predetermined vertex, visits the various vertices precisely once, closes at the same determined [4] [5].

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TSP is able to be utilized in a considerable measure of issue arrangements and some solutions are for example printed circuit assembling, time and employment scheduling of the machines, mechanical apply autonomy, logistic or occasion directing, determining bundle transfer course in computer networks, and air terminal flight booking. In a solution of TSP, there are two fundamental methodologies. It is moderate and mostly infeasible for more serious issue sizes in the primary tries to locate a perfect arrangement. Inside a sensible time and with no surety for an ideal arrangement the second one tries is to discover an answer.

The aim of this paper is to study the mathematical theories that are used in traveling salesman problem. This paper will show the mathematical theory that is used in Symmetric travelling salesman problem (sTSP) and Asymmetric travelling salesman problem (aTSP).

This paper is organized as background as described in Section II, Section III explains about methodology in mathematical theory that is involved in traveling salesman problem. In section IV, the examples of TSP are shown. The discussion is in section V. Finally, the conclusion is given in Section VI.

II. BACKGROUND

The TSP is a combinatorial problem [6] [4]. TSP is actually to find the shortest tour of a group of cities without visiting any town twice, but applicable and it infers the Hamiltonian cycle inside a weighted completely associated undirected graph. So, this is a problem of combinatorial graph search.

The standard TSP might be introduced mathematically as finding the Hamiltonian cycle of minimum of weight within a weighted are completely associated undirected graph G = (V,E) where the vertices exhibit the urban communities, the edges mean the intercity ways, and the weights of the edges represent the intercity distances [7]. The deterministic strategy to solve the TSP issue includes navigating all possible routes, assessing comparing tour distances and discovering the tour of insignificant distance. The total number of conceivable routes traversing n cities is *n!*. Furthermore, in this way, in instances of extensive values of *n* it becomes difficult to discover the expense of all tours in polynomial time [8].

One of the illustrations that utilizes the mathematical theory to calculate the minimum tour length of the Potts Neural Network, the calculation is based on an energy function like to the one used in the original work by Hopfield and Tank [9]. The algorithm is,

$$E = \sum_{ij} D_{ij} \sum_{a} S_{ia} S_{j(a+1)} - \frac{\beta}{2} \sum_{i} \sum_{a} S_{ia}^2 + \frac{\alpha}{2} \sum_{a} (\sum_{i} S_{ia})^2$$

In the above equation, the first term minimizes the tour length (D_{ij} is the intercity distance matrix), and the second and third terms guarantee that every city is visited precisely once. A noteworthy novel property is that the condition of,

$$\sum_{a} S_{ia} = 1$$

is constantly fulfilled where the dynamics is limited to a hyper plane instead of a hypercube. Thus, the relating mean field equations read,

$$V_{ia} = \frac{e^{U_{ia}}}{\sum_b e^{U_{ib}}}$$

Where $V_{ia} = < S_{ia} > T$ and the local fields U_{ia} are given by

$$U_{ia} = -\frac{1}{T} \frac{\partial E}{\partial V_{ia}}$$

The mean field equations are minimizing the free energy (F = E – TS) corresponding to E in the first equation. A pivotal parameter when solving the mean field equations and local field is the temperature T. It ought to be chosen in the region of the critical temperature T_c.

III. METHODOLOGY

i. General numerical depiction for the TSP

As per the meaning of the TSP, its mathematical depiction is as per the following:

$$\min \sum d_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1,2,\cdots n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1,2,\cdots n$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \quad 2 \leq |S| \leq n-2, S \subset \{1,2,\cdots n\}$$

$$x_{ij} \in \{0,1\} \quad i,j = 1,2,\cdots n \quad i \neq j$$

Figure 1: Formula, Matai, et al. [10]

ii. Formulation of sTSP

For definite examination, late overviews by Orman and Williams [11] can be alluded. One of the stand out amongst the most referred mathematical formulation for TSP are the Dantzig, et al. [12] formulation. The most performing careful calculation which currently accessible, was dispersed under the title ‘Implementing the Dantzig–Fulkerson–Johnson algorithm for large traveling salesman problems’ are actually an early portrayal of Concorde [13]. In each inverse course which framing an undirected graph, the separation between two urban areas is the same for symmetric TSP. This symmetry parts the quantity of conceivable solutions. The formulation associates a binary variable x_{ij} with every edge (i, j), must be equivalent to 1 if and only if the edge shows up in the ideal tour. The TSP definition is as per the following.

Minimize

$$\sum_{i<j} c_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{i<k} x_{ik} + \sum_{j>k} x_{kj} = 2 \qquad (k \in V) \tag{2}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \qquad (S \subset V, 3 \leq |S| \leq n-3) \tag{3}$$

$$x_{ij} = 0 \text{ or } 1 \qquad (i,j) \in E \tag{4}$$

Figure 2: Formula, Matai, et al. [10]

The above formulations are alluded to as degree limitations, sub tour disposal imperatives and internality requirements, respectively. Within the sight of (2), constraints (3) are logarithmically comparable to the network limitations

$$\sum_{i\in S, j\in V\setminus S, j\in S} x_{ij} \geq 2 \qquad (S\subset V, 3\leq |S| \leq n-3) \tag{5}$$

Figure 3: Formula, Matai, et al. [10]

iii. <u>Formulation of aTSP</u>

In the asymmetric case, the [12] formulation stretches out effortlessly to it. Ways may not exist in both course or the separations between two locations might be unmistakable, framing a coordinated diagram in the asymmetric TSP. Here x_{ij} is a binary variable, connected with curve (i,j) and equivalent to 1 if and only if the circular segment shows up in the ideal visit. The detailing is as per following.

Minimum

$$\sum_{i\neq j} c_{ij} x_{ij} \tag{6}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad (i\in V, i\neq j) \tag{7}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad (j\in V, j\neq i) \tag{8}$$

$$\sum_{i,j\in S} x_{ij} \leq |S|-1 \qquad (S\subset V, 2\leq |S| \leq n-2) \tag{9}$$

$$x_{ij} = 0 \text{ or } 1 \qquad (i,j)\in A \tag{10}$$

Figure 4: Formula, Matai, et al. [10]

IV. EXAMPLE IN TRAVELLING SALESMAN PROBLEM

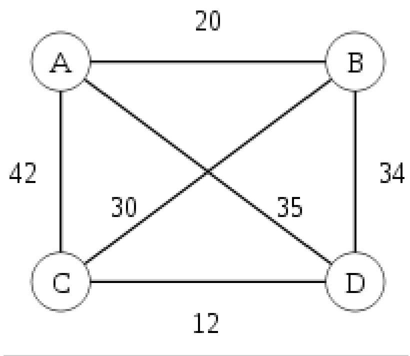

Figure 5: Symmetric TSP with four city, contributors [14]

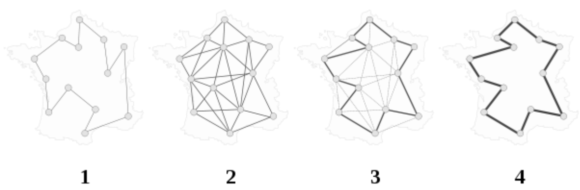

Figure 6: Ant Colony Optimization, contributors [14]

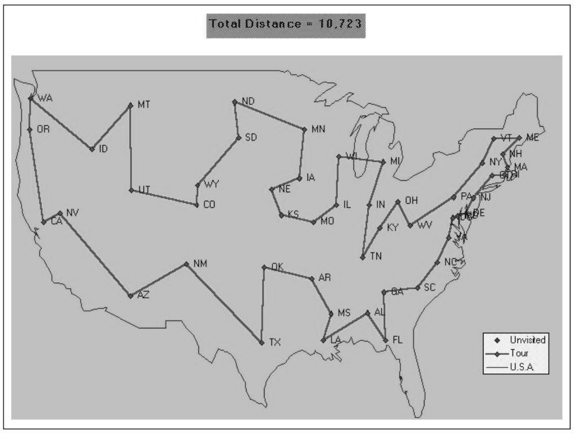

Figure 7: USA TSP map, Daskin [15]

V. DISCUSSION

Travelling salesman problem have been numerically reviewed in this paper about the mathematical theory that might be used in calculating to minimize the tour length. It has some direct applications to real-life problems, however it underlines many other optimization problem too. The mathematical theory that is shown in methodology can be used in example of printed circuit assembling, time and employment booking of the machines, mechanical apply self-rule, logistic or event steering, determining bundle transfers course in computer networks, and airplane terminal flight planning.

VI. CONCLUSION

The objective of this paper is to study and show the mathematical theories that are used in the traveling salesman problem. This paper has shown the mathematical theory that is used in symmetric Travelling Salesman Problem (sTSP) and asymmetric Travelling Salesman Problem (aTSP). So, it has demonstrated the mathematical theories used in calculating the tour length of symmetric TSP and asymmetric TSP.

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YOUTUBE LINK

https://www.youtube.com/watch?v=Ud6Y7VwuJmI

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