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Prim's Minimum Cost Spanning Tree Theory

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***Abstract*— This paper states the theory of the Prim's algorithm starting with the history of this algorithm. Then this paper provides an example of Prim's algorithm for a better understanding of this algorithm and minimum cost spanning tree. Beside that a few applications which were done by other researchers are included in this paper.**

***Keywords:* Prim's algorithm, minimum cost spanning tree**

I. INTRODUCTION

Prim's algorithm was discovered in 1930 by mathematician Vojtech Jarnik and later independently by computer scientist Robert C. Prim in 1957. The algorithm continuously increases the size of a tree starting with a single vertex until it spans all the vertices. Prims algorithm is faster on dense graphs. Prims algorithm runs in O(n*n) but the running time can be reduced using a simple binary heap data structure and an adjacency list representation.

Prims algorithm for finding a minimal spanning tree parallels closely the depth- and breadth-first traversal algorithms. Just as these algorithms maintained a closed list of nodes and the paths leading to them, Prims algorithm maintains a closed list of nodes and the edges that link them into the minimal spanning tree. Whereas the depth-first algorithm used a stack as its data structure to maintain the list of open nodes and the breadth-first traversal used a queue, Prims uses a priority queue.

Prim's algorithm is a greedy algorithm; it starts by selecting an arbitrary vertex as the root of the tree [13]. It then grows the tree by adding a vertex that is closest (has the shortest edge to) the current tree and adding the shortest edge from any vertex already in the tree to the new vertex. The algorithm terminates once all vertices have been added to the tree. The sum of all added edges is the cost of the minimum spanning tree, MST.

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Prim's algorithm is an algorithm in graph theory that finds a minimum spanning tree for a connected weighted graph [14]. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. If the graph is not connected, then it will only find a minimum spanning tree for one of the connected components.

II. MINIMUM SPANNING TREE

Minimum spanning tree (MST) is a classical problem in the graph theory. MST is used to find the shortest path using the Prim's algorithm approach. Minimum spanning trees is used for several reasons

They can quickly and easily compute the sub graph that reflects a lot of information about original graph. They provide a way to identify clusters in sets of points by deleting the long edges from a minimum.

As an educational tool, minimum spanning tree algorithms provide graphic evidence that greedy algorithms can give provably optimal solutions. In these classical algorithms Prim's algorithm is one of them. The serial computational complexity of Prim's algorithm implemented with traditional data structure is O (|V|2).

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The various connecting cities in Map with appropriate distances are identified and applying Prim's algorithms to find the shortest path based on the distance calculated from source to destination places. Finally, a graphical user interface has been developed to provide display and manipulation of the algorithm including calculation of cost based on the distance travelled.

III. PRIM'S ALGORITHM EXAMPLE

Consider the following graph and shows the various steps involved in the construction of the Minimum Cost Spanning Tree.

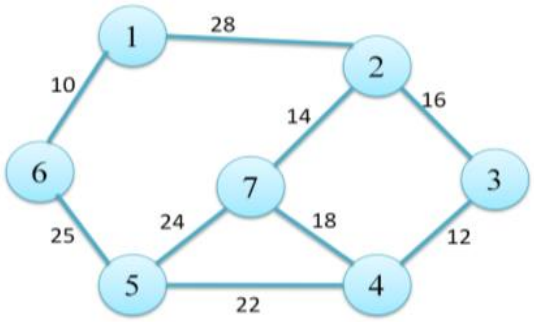

Figure 1. Connected graph

Based on the graph Prim's algorithm is used to find the shortest path to the destination. Connection is made from the start point which is node 1 to next node based on the lowest distance which is node 6. Then we proceed from node 6 to the next node repeating the steps until we reach node 7 which is the destination.

Figure 2 below shows the path to the destination using Prim's algorithm which is the shortest path we can get.

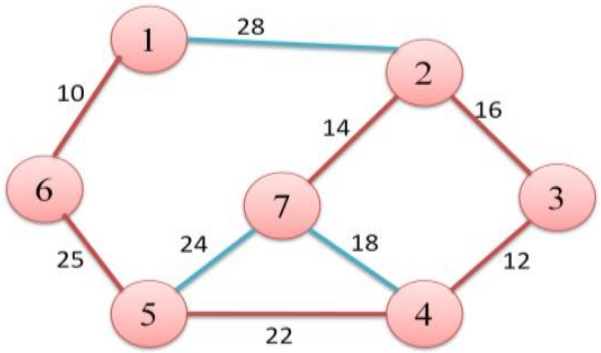

Figure 2. Completed Path

It found the destination using the Prim's algorithm and obtained the minimum cost which is 99 so the final path of minimum cost of spanning is {1, 6}, {6, 5}, {5, 4}, {4, 3}, {3, 2}, {2, 7}. The Prim's algorithm operates on two disjoint sets of edges in the graph. Prim's has a better running time if both the numbers of edges and the number of nodes are low.

IV. Literature Review

Prim's algorithm is used to find the distance between cities and for determining the shortest distance between two cities without having internet connectivity [14]. The installation of the software can be done in multimedia mobile phones or laptops. It is software in which an accurate distance with proper scaling measured. By simply choose the cities on the map and the distance between them. To make it user friendly, we use the graphics. By this concept a person can find shortest between the two cities.

Prim's algorithm is used to improve local access network in rural areas [13]. They create an application of Prim's algorithm to local access network in rural areas. The minimum spanning tree problem has important applications in network design which has been extensively studied in literature. The minimum spanning tree problem on a graph with edge costs and vertex profits asks for a sub tree maximizing the difference between the total cost of all edges in the sub tree and the total profits of all vertices contained in the sub tree. Minimum spanning tree problem appears in the design of utility networks (e.g. bus services, electrifications) where villages and the network connecting them have to be chosen in the most profitable way. The application of Prim's algorithms is demonstrated to the design of local access networks in a particular local government area of about 88 villages. The case study is analyzed, and interesting results are obtained. The results obtained justify the need to apply this kind of algorithm for efficiency and profitability.

Prim's algorithm is used to design and implement a GPU [5]. A minimum spanning tree algorithm using Prim's approach on Nvidia GPU under CUDA architecture. By using new developed GPU-based Min-Reduction data parallel Primitive in the key step of the algorithm, higher efficiency is achieved. Experimental results show that we obtain about 2 times speedup on Nvidia GTX260 GPU over the CPU implementation and 3 times speedup over non-Primitives GPU implementation.

V. Conclusion

It is found that Prim's algorithm is very effective in providing shortest distance between two places. In one way it reduces the cost of fuel and time for transportation of passengers from one town to another which normally determines prices in transportation. Therefore, this paper can serve as a basic knowledge and be further explored into advanced knowledge for effective shortest path calculation which is important for travel transportation. The method can also be explored by other organizations willing to achieve their goals with a reduced cost and effective use of their resources especially in areas of electrical installation, pipe water distribution and telecommunication.

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