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Edge Insertion Algorithm

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***Abstract*—A new paradigm that supports dynamical changes of the topology of polygonal meshes is introduced in this paper. Edge insertion operation will change the faces of the 2-manifold object represented by the graph rotation system in two cases. The two cases were discussed in this paper. This research use a Doubly Linked Face List (DLFL), a data structure that always corresponds to a graph rotation system to make sure it always in valid topological consistency.**

***Keywords*: edge, insertion, parallel, algorithm, Catmull-Clark**

I. INTRODUCTION

Edge insertion algorithm will reduce the CPU time used if it perform updates of the distance estimations only after performing many edge insertions and will also reduce the memory requirement when it computes the distance between objects on the fly. It produces good quality t-spanners (few edges) that will be used to predict expected number of edges per node in order to speed up other algorithms that rely on massive edge insertion [5].

Paper [6] proposed a new method for edge and vertex insertion on four-directional C1 quadratic box spline surfaces. Edge insertion treated as a decomposition problem. The subdivision rules along and close to edges and vertices, it was derived by using commutativity of knot insertion and subdivision. This algorithm was then proven as algorithm that achieves space containment.

A flexible concept of embedding constraints which allows to model a wide range of constraints on the order of incident edges was introduced in [8]. This is by considering the problem of finding a planar combinatorial embedding of a planar graph where an additional edge can be inserted with the minimum number of crossings. The results showed that optimal edge insertion can still be performed in linear time when embedding constraints have to be respected.

Research made by [9] gave a new algorithm for the problem of finding an upward planarization for graphs with directed and undirected edges. In undirected case, the edges which are not part of the planar subgraph in the first step can be inserted independently of each other. In directed case, an edge cannot be inserted into the drawing without looking at the remaining edges which have to be inserted later. When dummy nodes are introduced in the graph it changed the ordering of the vertices which introduces directed cycles if an edge is added later.

The paper formulates the edge insertion paradigm as a general method to compute optimal triangulations and identified two criteria for which the paradigm indeed finds the optimum [14]. Edge insertion iteratively improves a triangulation by adding a new edge, deleting old edges crossing the new edge, and retriangulating the polygional regions on side of new edge.

Paper [12] proposed a programming paradigm that gets close to both the semantic simplicity of relational algebra and the expressive power of unrestricted programming languages. This algorithm resembles Warshall's algorithm (node insertion) but with doing edge insertion. Warshall's algorithm runs in $O(n^3)$ time while the edge insertion algorithm runs in $O(n^2)$ time.

The other paper has obtained the effciency of the algorithm and the quality of the triangulations. An $O(n^2 \log n)$ time algorithm is fairly easy to implement and is based on the edge insertion scheme that iteratively improves an arbitrary initial triangulation. The implementation of the edge insertion algorithm of this paper minimizes the angle and much faster on

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the average than expressed by the worst case analysis [10].

A new paradigm that allows dynamically changes of the topology of 2-manifold polygonal meshes studied [1]. These topological changes are simply done by inserting edges to a polygonal mesh, proving that it is possible to change the topology for progressive meshes and multiresolution representations by including the edge insertion and operations. Thus, this approach may potentially add new power to the existing morphing, compression and simplification scheme.

The overall runtime of the optimal edge insertion algorithm is linear [11]. Thus, it will match the algorithm for the fixed embedding case. Paper [8] presented a class of almost planar graphs which optimal edge insertion does not lead to an approximation algorithm with a constant factor. On the other hand, paper [5] has shown that optimal edge insertion approximates the optimal number of crossings in almost planar graphs by a factor of the maximum node degree in the graph.

Paper [2] proposed a novel representation technique and suitable variation operators for the degree-constrained minimum spanning tree problem. In this way, the insertion or deletion of an edge and the test whether a given edge is contained in the tree or not can be performed with expected constant effort. The edge insertion mutation is based on the principle of inserting a randomly chosen new edge and deleting another edge lying on the cycle introduced by the insertions.

II. EDGE INSERTION IN GRAPH ROTATION SYSTEM

Any rotation order of two vertices of a graph rotation system can be given by the following representation without loss of generality:

$V_1 : V_3V_2\alpha \qquad V_4 : V_6V_5\beta$

where α and β are any sequence of vertices. Let an edge be inserted as shown in Figure 1, the new graph rotation system can easily be obtained by simply updating the rotation orders as follows:

$V_1 : V_3\ V_4\ V_2\alpha \qquad V_4 : V_6\ V_1\ V_5\beta$

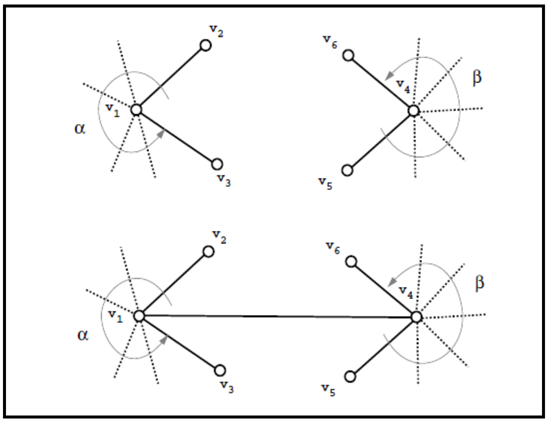

Figure 1. Inserting an edge [1]

Edge insertion operation will change the faces of the 2-manifold object represented by the graph rotation system. It either connects two faces into one single face or subdivides a single face into two faces. These two cases are illustrated in more detail in Figure 2 and Figure 3 below:

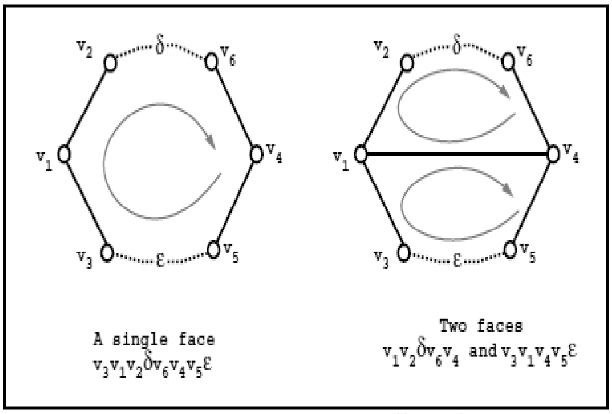

Figure 2. Inserting an edge between two different faces merges the two faces [1]

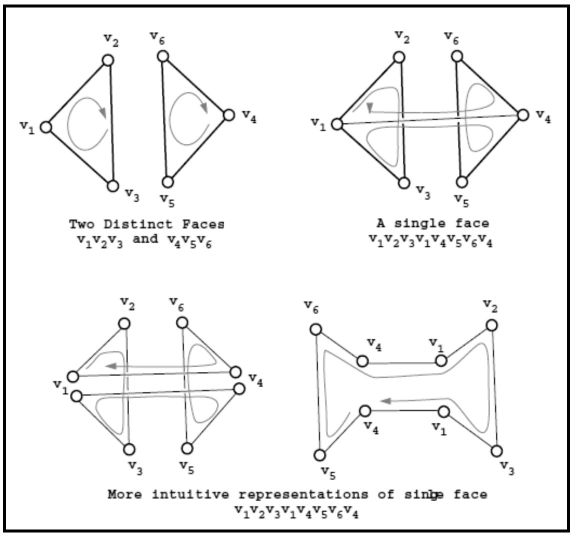

Figure 3. Inserting an edge between two vertices of the same face divides it into two faces [1]

III. IMPLEMENTATION

Rotation systems can be easily implemented as a set of linked lists. This structure is simple and easily implemented. The only practical is that it requires construction of faces in each step. Hence, Doubly Linked Face List (DLFL), a data structure that always corresponds to a graph rotation system was used since it is always in valid topological consistency. The main reason of using DLFL is that it supports logarithmic time edge insertion operations with face construction.

To show the feasibility of this approach, various 3D models that shows the creation of handles and holes were presented. In the examples, first irregular meshes was converted into meshes that consist only quadrilaterals to obtain initial mesh. Then, the initial mesh been smooth out by using Catmull-Clark subdivision scheme.

Figure 4 shows the connection of two disconnected toroids. This figure gives an example of handle improvement byinserting additional edges. As can be seen, when only one edge is inserted it creates a C1 discontinuous extraordinary vertex. Inserting extra edges removes this C1.

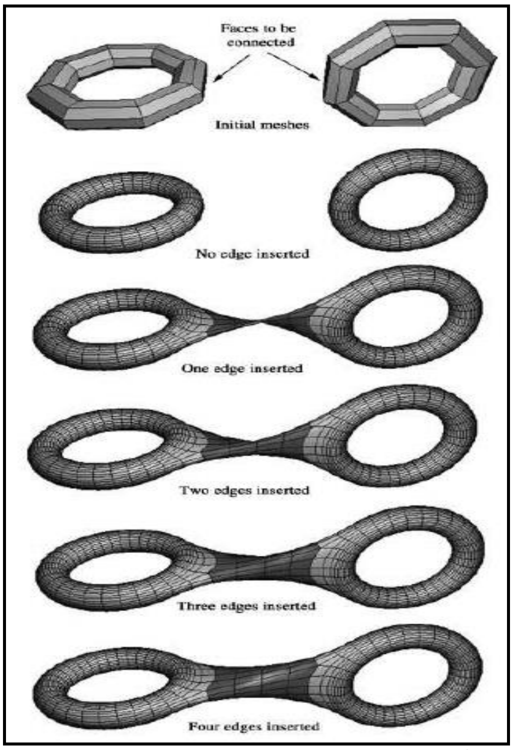

Figure 4. Connecting two toroids: the same colored faces connected with edge insertion [1]

Figure 5 demonstrates handle improvement by inserting additional edges.

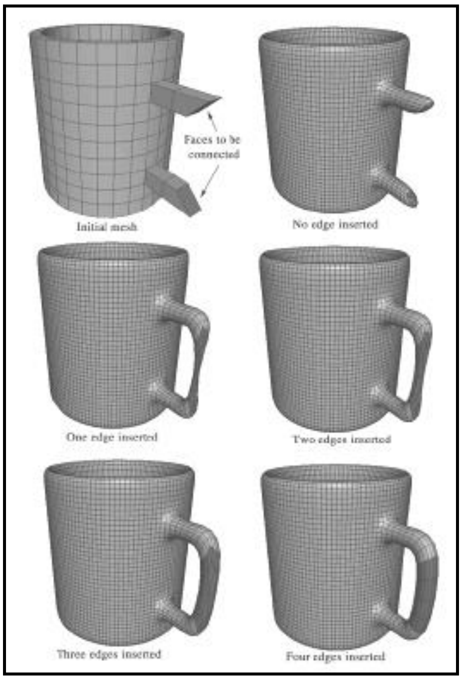

Figure 5. Creation of handle for a cup [1]

Figure 6 shows that each additional edge improves the quality of the hole in the cube.

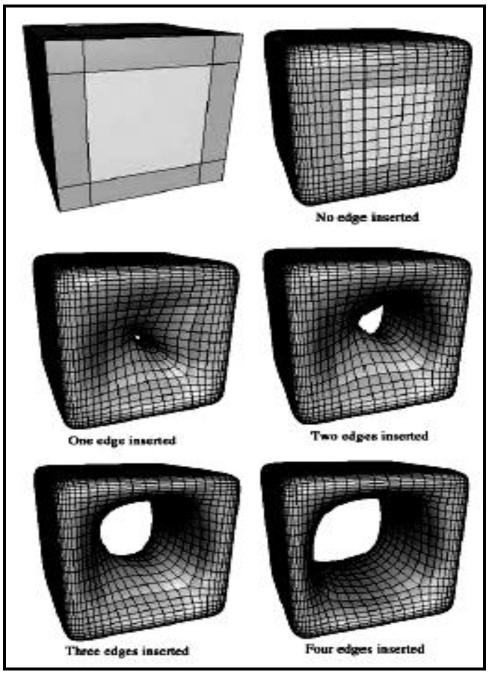

Figure 6. Creation of a hole in a cube [1]

IV. CONCLUSION

In this paper, a new paradigm is presented for changing topology of 2-manifold meshes without using an implicit approach. This new paradigm guarantees the 2-manifold property for meshes during these topological changes. Since rotation systems can easily be implemented as a set of linked lists, Doubly Linked Face List (DLFL) is found to have supports on logarithmic time edge insertion operations.

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REFERENCES

[1] E. Akleman, J. Chen, and V. Srinivasan, "A New Paradigm for Changing Topology of 2-Manifold Polygonal Meshes," *Technical Report TR00-011,* 2000.

[2] G. R. Raidl, "An Efficient Evolutionary Algorithm for the Degree-Constrained Minimum Spanning Tree Problem," *Facoritenstraße,* pp. 9-11, 1861.

[3] J. Monnot, "A note on the traveling salesman reoptimization problem under vertex insertion," *Information Processing Letters,* vol. 115, pp. 435-438, 2015.

[4] Q.-H. Hou, "An insertion algorithm and leaders of rooted trees," *European Journal of Combinatorics,* vol. 53, pp. 35-44, 2016.

[5] P. Hlineny and G. Salazar, *Lecture Notes in Computer Science* vol. 4372. Springer-Verlag, Berlin, Germany, 2007.

[6] A. Habib and J. Warren, "Edge and vertex insertion for a class of C1 subdivision surfaces," *Computer Aided Geometric Design,* vol. 16, pp. 223–247, 1999.

[7] C. Gutwenger, P. Mutzel, and R. Weiskircher, "Inserting an Edge into a Planar Graph," *Algorithmica,* vol. 41, pp. 289-308, 2005.

[8] C. Gutwenger, K. Klein, and P. Mutzel, "Planarity Testing and Optimal Edge Insertion with Embedding Constraints," pp. 126–137, 2007.

[9] M. Eiglsperger, F. Eppinger, and M. Kaufmann, "An Approach for Mixed Upward Planarization," *Journal of Graph Algorithms and Applications,* vol. 7, pp. 203-220, 2003.

[10] H. Edelsbrunner, T. S. Tan, and R. Waupotitsch, "An O(n2 Log n) Time Algorithm For The Minmax Angle Triangulation," *SIAM J, Scientifc & Statistical Computing,* vol. 13, pp. 994-1008, 1992.

[11] P. Eades, C. Gutwenger, S.-H. Hong, and P. Mutzel, *Algorithms and Theory of Computation Handbook, Second Edition*, Second ed. vol. 2: CRC Press.

[12] V. Breazu-Tannen, P. Buneman, and S. Naqvi, "Structural Recursion as a Query Language," *Logic & Computation,* vol. 46, 1992.

[13] C. Biernacki and J. Jacques, "A generative model for rank data based on insertion sort algorithm," *Computational Statistics and Data Analysis,* vol. 58, pp. 162-176, 2014.

[14] M. Ben, H. Edelsbrunner, D. Eppstein, S. Mitchell, and T. S. Tan, "Edge Insertion for Optimal Triangulations."

[15] C. Artigues and F. Roubellat, "A polynomial activity insertion algorithm in a multi-resource schedule with cumulative constraints and multiple modes," *European Journal of Operational Research,* vol. 127, p. 297-316, 2000.